JOURNAL OF COMPUTATIONAL PHYSICS 113, 1–4 (1994)

Effects of the Computational Time Step on Numerical Solutions of Turbulent Flow

HAECHEON CHOI* AND PARVIZ MOIN

Center for Turbulence Research, Stanford University, Stanford, California 94305 and NASA Ames Research Center, Moffett Field, California 94035

Received May 10, 1993

Effects of large computational time steps on the computed turbulence were investigated using a fully implicit method. In turbulent channel flow computations the largest computational time step in wall units which led to accurate prediction of turbulence statistics was determined. Turbulence fluctuations could not be sustained if the computational time step was near or larger than the Kolmogorov time scale. © 1994 Academic Press, Inc.

1. INTRODUCTION

All direct and large eddy turbulence simulations to date have either used explicit time advancement methods or semi-implicit methods (implicit treatment of viscous terms) for wall-bounded flows. Examination of the three-dimensional frequency/wave-number power spectrum of wall-pressure fluctuations in turbulent channel flow obtained from direct numerical simulations [1] reveals that negligible power resides in or about the Nyquist frequency corresponding to the computational time step used with a commonly used semi-implicit method. The computational time step used in the channel simulations of Kim *et al.* [2] was $0.0676\nu/u_\tau^2$, where ν is the kinematic viscosity and u_τ is the wall-shear velocity. Since the viscous time scale in the sublayer is $O(1)$ in wall units [3], one may conclude from this observation that the restriction on time step imposed by semi-implicit methods may be too stringent to maintain accuracy. Furthermore, complex geometries and the associated grid distributions and clusterings may impose a severe restriction on the time step for direct simulation of complex turbulent flows. For example, if the flow geometry contains sharp corners (e.g., riblet tips), rapid variation of flow variables in their vicinity requires dense grid clustering which would restrict the computational time step. A fully implicit method is required to determine the maximum time step that can be taken while maintaining accuracy.

A fully implicit method for the unsteady incompressible Navier–Stokes equations was developed in generalized coordinate systems (see Choi *et al.* [4]). This method is based on a fractional-step technique [5] in conjunction with a Newton-iterative scheme for solving the nonlinear momentum equations. The flow field is represented on a staggered grid, and a Poisson equation for the pressure correction is solved to satisfy the continuity equation at every time step.

The objectives of the present study are to investigate the effect of the computational time step on turbulence statistics using a fully implicit method and to find the largest computational time step in wall units with which accurate prediction of turbulence statistics in plane channel flow can be obtained at a given Reynolds number.

The numerical method is briefly described in Section 2. Numerical results for turbulent plane channel flow with different computational time steps are presented in Section 3, followed by a summary in Section 4.

2. NUMERICAL METHOD

The governing equations for an incompressible flow are

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_i u_j = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} u_i, \qquad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0, \qquad (2)$$

where x_i's are the Cartesian coordinates, and u_i's are the corresponding velocity components. All variables are non-dimensionalized by a characteristic velocity and length scale, and Re is the Reynolds number.

The integration method used to solve Eqs. (1) and (2) is based on a fully implicit, fractional step method; all terms in Eq. (1) are advanced with the Crank–Nicolson method in

* Permanent address: Department of Mechanical Engineering, Seoul National University, Seoul 151-742, Korea.

0021-9991/94 $6.00
Copyright © 1994 by Academic Press, Inc.
All rights of reproduction in any form reserved.

time. A four-step time advancement scheme for Eqs. (1) and (2) is

$$\frac{\hat{u}_i - u_i^n}{\Delta t} + \frac{1}{2}\frac{\partial}{\partial x_j}(\hat{u}_i\hat{u}_j + u_i^n u_j^n) = -\frac{\partial p^n}{\partial x_i} + \frac{1}{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}(\hat{u}_i + u_i^n), \quad (3)$$

$$\frac{u_i^* - \hat{u}_i}{\Delta t} = \frac{\partial p^n}{\partial x_i}, \quad (4)$$

$$\frac{\partial}{\partial x_i}\frac{\partial p^{n+1}}{\partial x_i} = \frac{1}{\Delta t}\frac{\partial u_i^*}{\partial x_i}, \quad (5)$$

$$\frac{u_i^{n+1} - u_i^*}{\Delta t} = -\frac{\partial p^{n+1}}{\partial x_i}. \quad (6)$$

Implicit treatment of the convective and viscous terms eliminates the numerical stability restriction. By substituting Eqs. (4) and (6) into Eq. (3), one can easily show that the overall accuracy of the splitting method is second order. Note that the present scheme does not require any special treatment of the intermediate velocity boundary condition (see, for example, Kim and Moin [5] about this issue). That is, the intermediate velocity boundary condition is simply $\hat{u}_i = u_i^{n+1}$ to second order in the time step. This can be shown from Eqs. (4) and (6),

$$\hat{u}_i = u_i^{n+1} + \Delta t \frac{\partial(p^{n+1} - p^n)}{\partial x_i} = u_i^{n+1} + O(\Delta t^2). \quad (7)$$

All the spatial derivatives are resolved with the second-order central-difference scheme using a staggered mesh system. The discretized nonlinear momentum equations are solved using a Newton-iterative method. The continuity equation is satisfied through the solution of a Poisson equation (5). A complete description of a variant of this method in generalized coordinate systems is given in Choi *et al.* [4].

3. NUMERICAL RESULTS FOR TURBULENT PLANE CHANNEL FLOW

The computations were carried out for a Reynolds number of 4200 based on the *laminar* centerline velocity U_l and the channel half-width δ which corresponds to a Reynolds number of about 180, based on the turbulent wall-shear velocity u_τ and the channel half-width δ. For the Reynolds number considered here, the computational box is chosen to be the *minimal flow unit* of Jiménez and Moin [6]; the streamwise and spanwise computational periods are $\pi\delta$ and $0.289\pi\delta$, respectively (roughly 570 and 160 wall units, respectively). The grid points used are $16 \times 129 \times 32$ in the x, y, and z directions, respectively. Uniform meshes with spacing $\Delta x^+ = \Delta x u_\tau/\nu \approx 35$ and $\Delta z^+ = \Delta z u_\tau/\nu \approx 5$ are used in the streamwise and spanwise directions. A non-uniform mesh of 129 points with hyperbolic tangent distribution is used in the wall-normal direction. The first mesh point away from the wall is at $y^+ = y u_\tau/\nu \approx 0.15$, and the maximum spacing (at the centerline of the channel) is seven wall units.

The initial flow field is an instantaneous solution of the Navier–Stokes equations previously obtained using a semi-implicit method. Starting from this initial velocity field, the governing equations were integrated forward in time until the numerical solutions reached statistically steady states. These statistically steady states were identified by a quasi-periodic behavior of the wall-shear stresses. Once the velocity field reached the statistically steady state, the equations were integrated further in time to obtain time averages of the various statistical quantities. The total averaging time was about $500\ \delta/U_l$ ($\approx 4000\nu/u_\tau^2$).

Six different computational time steps in wall units, $\Delta t^+ = \Delta t u_\tau^2/\nu = 0.2$, 0.4, 0.8, 1.2, 1.6, and 2, have been investigated. These computational time steps correspond to the CFL numbers, $\mathrm{CFL} = \max(|u|/\Delta x + |v|/\Delta y + |w|/\Delta z)\,\Delta t = 0.5$, 1, 2, 3, 4, and 5.

The time histories of the plane-averaged wall-shear rates for four different computational time steps are shown in Fig. 1. Stochastic and intermittent behavior of the wall shear stress is clearly discernible for the small computational time steps. It is interesting to note that the calculations with $\Delta t^+ = 1.6$ and 2 resulted in *laminar* flow solutions. The viscous time scale in the sublayer (the Kolmogorov time scale), $\tau^+ \equiv (u_\tau^4/\varepsilon\nu)^{1/2}$ [3], is about 2.4 in wall units, where ε is the dissipation rate per unit mass (the profile of ε for turbulent channel flow is shown in Mansour *et al.* [7]). Given that the computational time steps of 1.6 and 2 are close to the Kolmogorov time scale, it may be deduced from this observation that the computational time

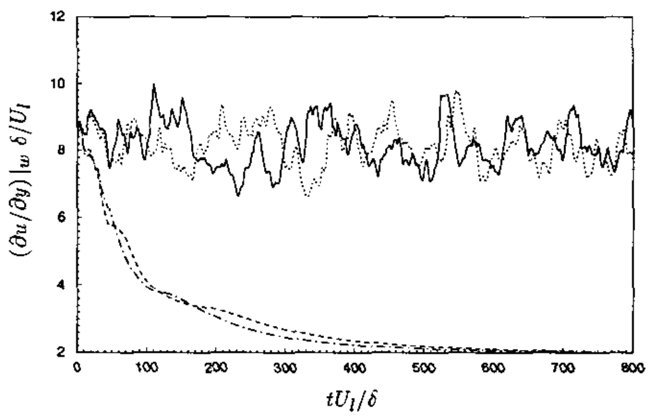

FIG. 1. Time histories of the plane-averaged wall-shear rates for different computational time steps: ——, $\Delta t^+ = 0.2$; $\cdots$, $\Delta t^+ = 0.4$; ---, $\Delta t^+ = 1.6$; -·-, $\Delta t^+ = 2.0$. Normalized wall shear rate $(\partial u/\partial y)|_w\,\delta/U_l = 2$ corresponds to fully developed laminar plane channel flow.

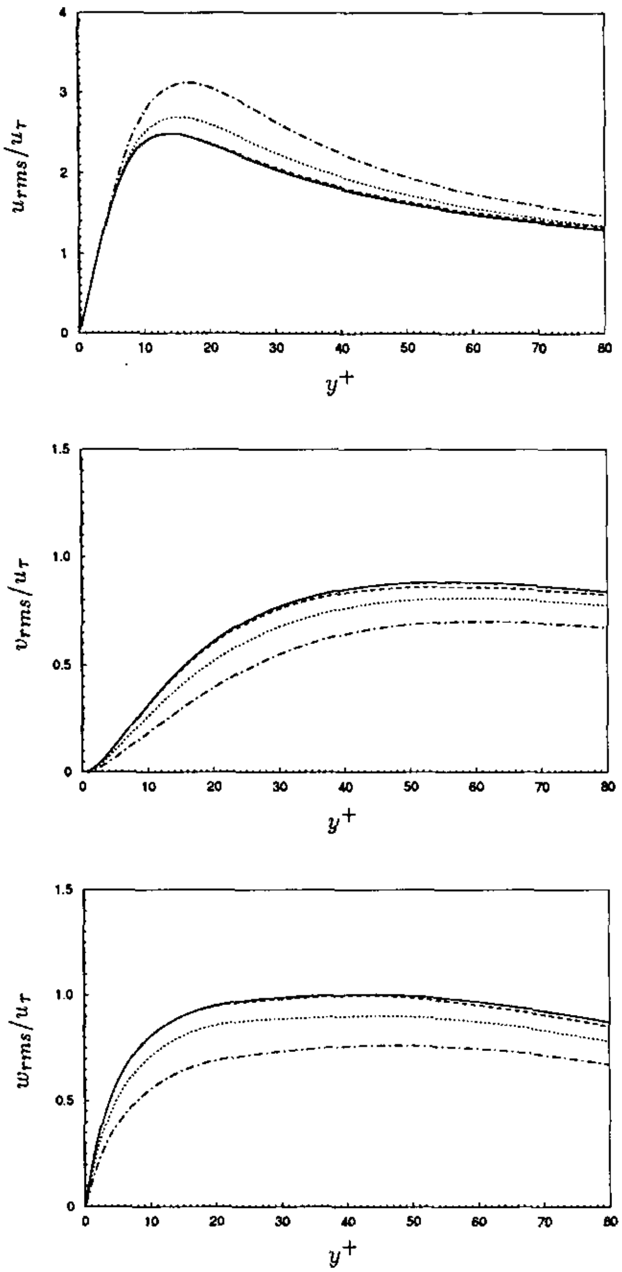

FIG. 2. Variation of root-mean-square velocity fluctuations with the computatonal time step: ——, $\Delta t^+ = 0.2$; – – –, $\Delta t^+ = 0.4$; $\cdots$, $\Delta t^+ = 0.8$; – · –, $\Delta t^+ = 1.2$.

step should be less than the Kolmogorov time scale to maintain turbulence.

It is well known that the amplification factor of the Crank–Nicolson scheme approaches -1 when applied to the diffusion equation with a large time step. This may then lead to unphysical effects on the turbulent motions (e.g., large dissipation) and one may conclude that the observed laminarizations with large time steps can be attributed to the unphysical behavior of the numerical scheme rather than to the poor resolution of the time scales of turbulent eddies. Therefore, the calculations with $\Delta t^+ = 1.6$ and 2 were repeated with the backward Euler scheme applied to the diffusion terms; however, they once again resulted in laminar flow solutions, confirming the Crank–Nicolson results.

Figure 2 shows the turbulence intensities normalized by the wall shear velocity. The results using the present fully implicit method are shown only for $\Delta t^+ = 0.2$, 0.4, 0.8, and 1.2. The calculations with large computational time steps overpredict the steamwise fluctuations ($u_{\rm rms}$) and underpredict the normal and spanwise fluctuations ($v_{\rm rms}$, $w_{\rm rms}$) when compared with the result of the smallest computational time step ($\Delta t^+ = 0.2$). The turbulence intensity profiles from the simulation with $\Delta t^+ = 0.4$ nearly coincide with those of the $\Delta t^+ = 0.2$ simulation.

The Reynolds shear stress profile is shown in Fig. 3. The computations with large computational time steps underpredict the peak Reynolds shear stress. The Reynolds shear stress from the $\Delta t^+ = 0.4$ calculation is very close to that from the $\Delta t^+ = 0.2$ calculation.

Vorticity fluctuations have significant contributions from small-scale turbulence motions. Root-mean-square vorticity fluctuations normalized by the mean shear at the wall ($\omega_i \nu / u_\tau^2$) are shown in Fig. 4. The calculations with large

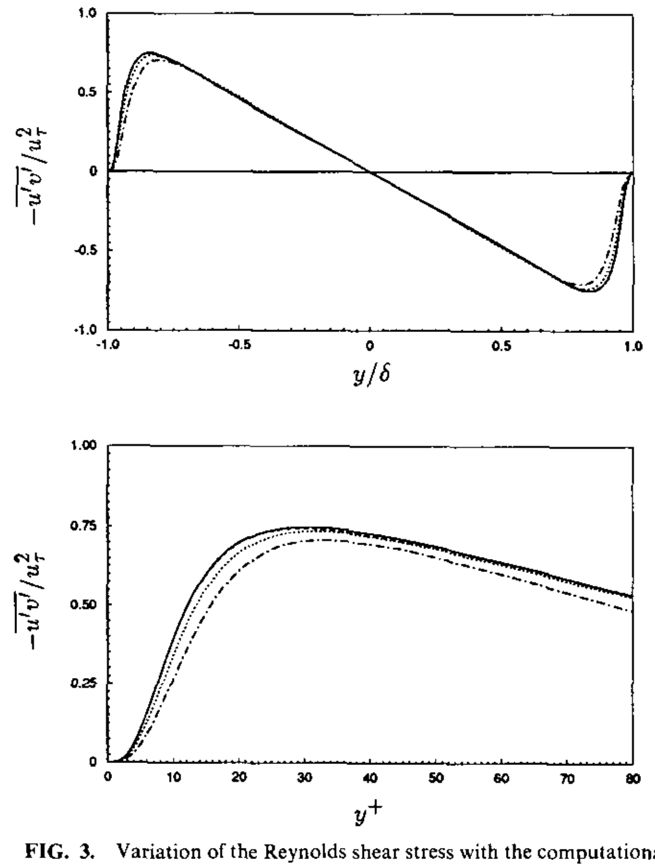

FIG. 3. Variation of the Reynolds shear stress with the computational time step: ——, $\Delta t^+ = 0.2$; – – –, $\Delta t^+ = 0.4$; $\cdots$, $\Delta t^+ = 0.8$; – · –, $\Delta t^+ = 1.2$.

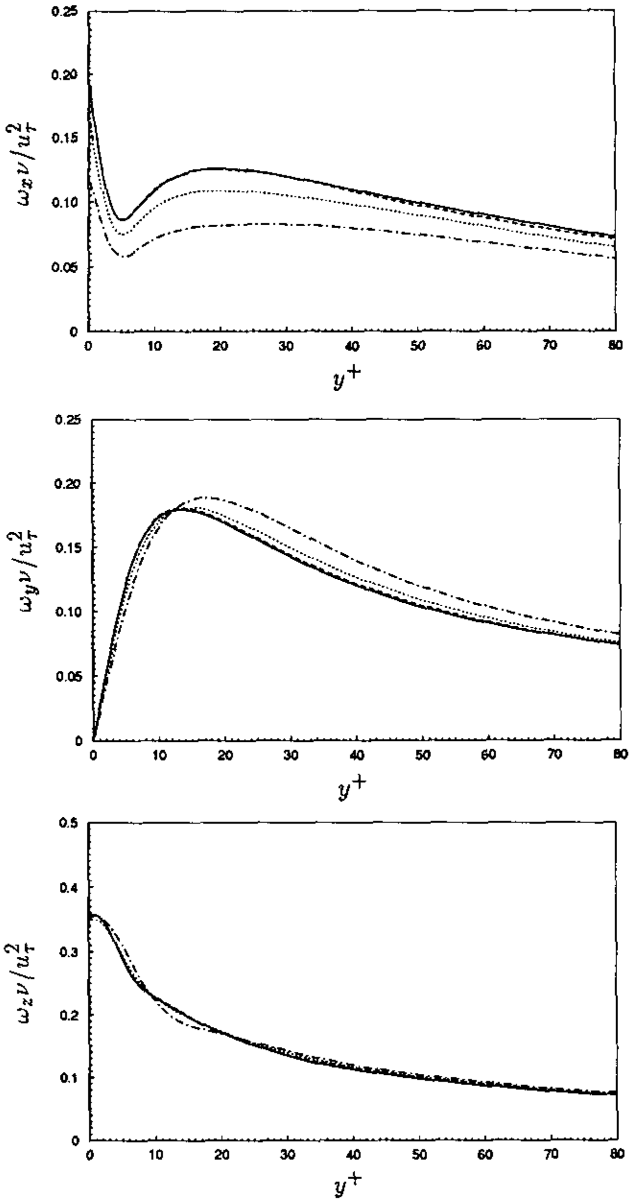

FIG. 4. Variation of root-mean-square vorticity fluctuations with the computational time step: ——, $\Delta t^+ = 0.2$; – – –, $\Delta t^+ = 0.4$; · · ·, $\Delta t^+ = 0.8$; – · –, $\Delta t^+ = 1.2$.

computational time steps underpredict the streamwise vorticity fluctuations and overpredict the normal vorticity fluctuations when compared with the results of the computations with $\Delta t^+ = 0.2$. The vorticity fluctuations from the $\Delta t^+ = 0.4$ and $\Delta t^+ = 0.2$ computations are nearly identical.

It appears that turbulence statistics from simulations with $\Delta t^+ = 0.4$ ($\Delta t U_l / \delta = 0.05$) are sufficiently close to those from calculations with $\Delta t^+ = 0.2$. The non-dimensional computational time step $\Delta t^+ = 0.4$ was therefore used in the computation of turbulent flow over riblets (Choi *et al.* [8]), where application of a variant of the present fully implicit method in curvilinear coordinates led to a factor of five savings in the required CPU time as compared to a semi-implicit method. Note that in the riblet computations, $\Delta t^+ = 0.4$ corresponded to CFL ≈ 3.

4. SUMMARY

The effect of large computational time steps on the computed turbulence was investigated for the first time using a time-accurate, fully implicit method. The objective was to find the largest time step in wall units which accurately predicts turbulence statistics in turbulent plane channel flow at a given Reynolds number. It was demonstrated that turbulence fluctuations can only be sustained if the computational time step is appreciably less than the Kolmogorov time scale. Application of a fully implicit method to turbulent flow over riblets resulted in a factor of five savings in the required CPU time compared to a semi-implicit method.

ACKNOWLEDGMENTS

We are grateful for useful discussions with Drs. John Kim and Hans-Jakob Kaltenbach. Financial support from the Air Force Office of Scientific Research under Contract No. F49620-93-1-0078 is gratefully acknowledged.

REFERENCES

1. H. Choi and P. Moin, *Phys. Fluids A* **2**, 1450 (1990).
2. J. Kim, P. Moin, and R. Moser, *J. Fluid Mech.* **177**, 133 (1987).
3. H. Tennekes and J. L. Lumley, *A First Course in Turbulence* (MIT Press, Cambridge, MA, 1972).
4. H. Choi, P. Moin, and J. Kim, Report No. TF-55, Department of Mechanical Engineering, Stanford University, Stanford, CA, 1992.
5. J. Kim and P. Moin, *J. Comput. Phys.* **59**, 308 (1985).
6. J. Jiménez and P. Moin, *J. Fluid Mech.* **225**, 213 (1991).
7. N. N. Mansour, J. Kim, and P. Moin, *J. Fluid Mech.* **194**, 15 (1988).
8. H. Choi, P. Moin, and J. Kim, *J. Fluid Mech.* **255**, 503 (1993).